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A Survey of Structural Models of Corporate Debt Pricing

João C. A. Teixeira

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João C. A. Teixeira
Universidade dos Açores (DEG)
e CEEAplA

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RESUMO/ABSTRACT

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Keywords: structural models, corporate debt valuation, empirical credit spreads

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João C. A. Teixeira
Universidade dos Açores
Departamento de Economia e Gestão
Rua da Mãe de Deus, 58
9501-801 Ponta Delgada

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João C. A. Teixeira

University of the Azores - Department of Economics and Business
CEEAplA
Rua Mãe de Deus, s/n
9501-801 Ponta Delgada, Portugal

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Abstract

This paper surveys the theoretical and empirical literature on structural models of corporate debt pricing. It provides an understanding of the importance of structural models in predicting credit spreads, and focuses on the role of rating, maturity, asset volatility and sector effects.

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1. Introduction

During the last years we saw many theoretical developments in the field of credit risk research. Most of this research concentrated on the pricing of corporate and sovereign defaultable bonds as the basis of credit risk pricing. These studies can be divided in two main categories: structural models and reduced-form models[1].

Structural models have its origins in Merton (1974) framework, which has been the key foundation of corporate debt pricing. Relying on the contingent claims analysis of Black and Scholes (1973), Merton (1974) presents a simplified model that can be used to value each component of the firm’s liability mix. As noticed by Cossin and Pirrote (2001), “It is called the ‘structural approach’ because it relies entirely upon the sharing rule for the value of the assets of the firm between two main classes of claimholders, the shareholders and the bondholders, in other words, it depends on the actual capital structure of the firm”(p.15). In such a framework, the default process of a company is driven by the value of the company’s assets and the firm’s default risk is explicitly linked to the variability in the firm’s asset value. Under these structural models, all the relevant credit risk elements, including default, are a function of the structural characteristics of the firm: asset volatility (business risk) and leverage (financial risk).

Reduced-form models, on the other hand, do not condition default on the value of the firm, and parameters related to the firm’s value do not need to be estimated. Moreover, reduced-form models introduce explicit assumptions regarding the dynamics of default variables. These variables are modeled independently from the structural features of the firm, its asset volatility and leverage.

Built on the arbitrage-free methodology, the Merton (1974) model allows for the valuation of a firm’s debt and equity without a prior knowledge of the real drift of the firm’s asset. In some sense Merton (1974) expands the advantage of Black and Scholes (1973) framework to the valuation of a firm’s claims.

Despite this innovative nature, Merton (1974) model presents many shortcomings that are essentially due to its simplifying assumptions about reality. It assumes that the liability structure of the firm consists only of a single class of debt, a non-callable zero

[1] Schmid (2004) uses an alternative denomination for these categories. He classifies structural models as asset based models and reduced models as intensity based models.

coupon bond, and that bankruptcy is not only costless but also cannot be triggered before maturity. In addition, it assumes that the absolute priority rule always holds at maturity, meaning that equityholders can only obtain a positive payoff after debtholders being totally reimbursed. This is clearly an unrealistic assumption. Franks and Torous (1994) show that the strict absolute priority rule was violated in 78% of the bankruptcies of their sample. Another important stylised version of reality is the assumption of a flat term structure of interest rates.

Many papers, including Black and Cox (1976), Geske (1977), Leland (1994), Leland and Toft (1996), Longstaff and Schwartz (1995), Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997), Fan and Sundaresan (2000) and Collin-Dufresne and Golstein (2001) have extended the original Merton (1974) model to incorporate more realistic assumptions. A new assumption, which is common to all these models, and represents a major improvement of the Merton framework, is the possibility of early default. In these models the firm can go into bankruptcy before maturity, as soon as a bankruptcy trigger for the asset value is reached. Schmid (2004) classifies the models with this feature as First Passage Time Models.

Black and Cox (1976) is the first model to introduce net worth covenants, which provide the bondholders the right to force the firm into bankruptcy as the firm value hits some deterministic time-dependent threshold. Having triggered this level, bondholders receive the assets of the firm. Black and Cox also discuss the implications of different debt classes (senior and junior debt) and endogenize the default boundary. This model serves as the basis for many recent extensions of the structural approach. Even with this improvement, the pricing in this model continues to be done on zero-coupon bonds with zero recovery upon default. Moreover, it does not avoid the problem found in the Merton (1974) model of extreme underestimation of credit spread for very short maturities. In these two models, as maturity goes to zero, the spreads also go to zero.

One year later, Geske (1977) launches the design of equity as a compound option. He assumes that the stockholders of a company (with risky coupon bonds) have, at each coupon payment, the option to pay or not pay later coupons. This introduces the idea that stockholders have to take into account future debt obligations when deciding on the debt service. In this framework, bankruptcy occurs when the value of assets is so low that equityholders no longer find profitable to service debt. Equity and debt valuation requires the use of compound option theory.

Despite Black and Cox's (1976) introduction of early default and Geske's (1977) idea of equity as a compound option, these models could not present a solution to one of the main challenges of financial theory: the determination of an optimal debt policy. Regarding this question, they found a similar answer as Merton (1974) did: there is no optimal capital structure. Actually, this proposition had been introduced in the 1950's in the seminal paper of Modigliani and Miller (1958). Assuming frictionless markets and through an arbitrage argument Modigliani and Miller show that the value of the firm is not affected by the mix of equity and debt. The value of the firm depends only on the investment decisions and there is no role for the capital structure policy.

It was not until Leland's (1994) work that the optimal capital structure decision was operationalised in continuous time. Prior to Leland (1994), there was just a discrete time implementation of the capital structure decision by Kraus and Litzenberger (1973). Other authors have also discussed why a certain capital structure may be better than another, but just in a conceptual basis. Jensen and Meckling (1976) discuss the consideration of taxes, bankruptcy costs and agency problems (under investment, over investment and free cash flow) in the capital structure decision. Assuming that leverage causes potential conflicts of interest between equityholders, managers and debtholders, they establish conceptually an optimal capital structure as the one that minimizes the sum of agency costs of equity and agency cost of debt.

Leland (1994) extends Black and Cox (1976) endogenous default model to include the tax advantage of debt and bankruptcy costs. The first "real world friction" works an incentive to increase the leverage (because of the tax benefit of interest payment) and bankruptcy costs as a disincentive. The optimal capital structure decision is therefore a tradeoff between tax benefits and bankruptcy costs.

Similarly to Black and Cox (1976), Leland (1994) found that the default boundary increases with the coupon payment and decreases with the risk free rate and asset volatility. The only difference between the default boundaries in these two models is that the coupon has to be adjusted for the tax shield in Leland.

In deriving his results, Leland (1994) makes several assumptions. One of them, the infinite maturity debt, is relaxed two years later in Leland and Toft (1996), allowing the study of both the influence of debt amount on the capital structure decision (and credit spread), as well as the impact of debt maturity chosen. In this framework, the pricing formulas are no longer time independent as they were in Black and Cox (1976) and Leland (1994). In Leland and Toft (1996) we found many of the results obtained in previous structural models regarding the term structure of credit spread. The hump-shaped term structure of credit spreads on high levered firms and a monotonously increasing credit spread in low levered firms is confirmed. As in the Merton (1974) model, they found that credit spreads might not always increase with the maturity of the bonds, especially if we consider junk bonds.

Both Leland (1994) and Leland and Toft (1996) models assume that default occurs as soon as the equity value reaches zero. This is a strict consequence of the assumption that raising cash through an equity issue is costless and that the absolute priority rule is respected. However, deviations from the absolute priority rule are common. We may have the equityholders obtaining a positive value in the event of default even when senior claimers are not fully paid off. Moreover, bankruptcy procedures leave some considerable scope for strategic behaviours from the different claimants involved. The recognition of these "real world features" lead to the appearance of new structural models usually denominated strategic debt service models. These include Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997) and Fan and Sundaresan (2000) models.

Anderson and Sundaresan (1996) model the asset value as a binomial process and assume that this asset realizes a certain cash flow over time. Debtholders receive the contractual debt service as soon as the generated cash flow is sufficient to cover it. If the manager or owner of the firm defaults on the debt contract then debtholders receive the asset value less the amount of liquidation costs. The existence of these liquidation costs leaves some role for debt renegotiation between both parts. Anderson and Sundaresan (1996) assume that managers have all the bargaining power, making it possible for them to make take-it-or-leave-it offers to debtholders. With this power, managers will choose a debt service that cannot be higher than the cash flow of the firm. In the case that this debt service is equal to the contractual debt service, the firm continues to operate normally.

Otherwise, debtholders can choose whether they want to force the company into bankruptcy or accept a lower coupon.

Fan and Sundaresan (2000) enhance Anderson and Sundaresan's (1996) model by considering first, a continuous time-framework, and secondly, corporate taxes. It addition, they introduce a bargaining power parameter, making possible a redistribution of power between debtholders and equityholders. In Anderson and Sundaresan (1996) all the bargaining power is attributed to managers, who act in the equityholders's interest.

While the studies of Anderson and Sundaresan (1996) and Fan and Sundaresan (2000) assume a geometric Brownian motion process for the asset value of the firm, Mella-Barral and Perraudin (1997) assume an identical process for the output price of the firm's product. They further assume a fixed cost of operation. If new owners operate the firm after bankruptcy, there is a loss of efficiency as they can only operate the output with lower prices and higher costs. Alternatively, the firm can be liquidated for a certain liquidation value.

Using a time independent framework they found closed-form solution for the pricing of debt. In this pricing exercise the liquidation threshold plays an important role. Under a lower critical level for the output price, the firm is liquidated and bondholders receive the collateral. Above the upper critical level the debtholder receives its regular coupons. The debt renegotiation will occur whenever the state variable is between these two critical levels. If equityholders have the all bargaining power and thus can make take-it-or-leave-it offers to bondholders they propose coupon payments below the contractual coupon, approximating the debt value to the firm's liquidation value. The firm is operated by the original equityholders in a fully efficient way. On the other hand, if debtholders have all the bargaining power they will operate the firm but in a less efficient way than would occur without renegotiations. Debtholders will keep the firm alive by injecting capital until the liquidation is efficient.

Even though all these strategic debt service models have the advantage of incorporating deviations from the absolute priority rule, they rely on some simplifying assumptions about debt characteristics. They have the disadvantage of assuming perpetual coupon debt, making unfeasible the analysis of the term structure of credit spread. In addition, Anderson and Sundaresan (1996) cannot yield closed-form solutions for the pricing of bonds.

One of the main weaknesses of the Merton (1974) study and many other models that extended his work is the assumption of a flat term structure. This assumption contradicts what has been observed in most economies. In the last decades we have observed a mix of downward and upward sloping term structures, with the market prices of corporate bonds being sometimes highly sensitive to the slope of the term structure, rather then just the level. Therefore, it seems important to consider interest rate risk when valuing risky debt.

Kim *at al* (1993), Nielsen *et al* (1993) and Longstaff and Schwartz (1995) are the main contribution is this area. Among these, Longstaff and Schwartz (1995) model is the one that has deserved more attention in recent empirical papers. Recognising the shortcoming of the Merton model, that default can occur only at maturity, Longstaff and Schwartz (1995) also allow for early default. They model default as the time when the value of the debt reaches some constant threshold, which serves as a distress boundary. When the value of the assets reaches this barrier, default is triggered, and some form of restructuring occurs such that the remaining asset value is allocated among the firm's claimants. Hence, contrary to Merton (1974) and Black and Cox (1976), Longstaff and Schwartz (1995) assume that the allocation of the firm's assets is given exogenously.

Another improvement of Merton (1974) framework, proposed by Zhou (2001), is the assumption of a jump diffusion process where the firm can suddenly default because of a downward drop in its value. Under the traditional diffusion approach (GBM) this is not possible, implying that perfectly healthy firms have a null probability of bankruptcy and a corresponding credit spread of zero, while there is evidence of a systematic positive credit spread. With a jump-diffusion process it is possible to capture the jump risk in credit spread and fit a wide variety of term structures of credit spreads: flat, upward, downward sloping or even hump-shaped.

Several empirical studies have pointed out the weaknesses of the Merton (1974) model, in particular its incapacity to generate the levels of yields spreads observed in the market. These include, among others, the papers of Jones *et al* (1984), Ogden (1987), Wei and Guo (1997), Lyden and Saraniti, (2000), Ericsson and Reneby (2002) and Eom *et al* (2004).

Jones *et al* (1984) analysed 177 bonds issued by 15 firms and found that Merton model overestimates bond prices by an average of 4.5%. They conclude that the model performs better for speculative grade bonds and that prediction errors are systematically related to maturity, equity variance and leverage. Ogden (1987), on the other hand, looked at 57 callable bonds and sinkable corporate bonds and found that Merton model underestimates spreads by 104 basis points (bp) on average. Both studies conclude that the incorporation of a stochastic interest rate process may yield significant improvements in the performance of the model. These studies suffer from some problems with the inclusion of callable bonds and sinking fund provisions. By considering bonds with these features it is difficult to evaluate whether the underestimation revealed by the Merton model is due to its assumptions or to the pricing of these features by investors.

During this decade, the studies of Lyden and Saraniti (2000), Ericsson and Reneby (2002) and Eom *et al* (2004) represent an improvement in terms of the quality of the bond sample. All these studies not only use firms with simple capital structures but also exclude from the sample bonds with any call or sinking fund provision. Lyden and Saraniti (2000), who compare the performance of Merton (1974) model with the Longstaff and Schwartz (1995) model, find that both these models underestimate the credit spread. For the Merton model the average underestimation in credit spread is between 80 and 90 bp and the errors are systematically related to coupon and time to maturity. The allowance for early default and stochastic interest rate of the Longstaff and Schwartz model does not improve the performance of the model.

Ericsson and Reneby (2002), who implemented a perpetual bond model based in Black and Cox (1976) framework, found a good performance of the model. They also found that prediction errors are linked to liquidity. There is a greater underestimation of credit spread for speculative grade bonds, which are perceived to be less liquid.

To date, the most comprehensive empirical study about the performance of corporate debt pricing models is found in Eom *et al* (2004). They assess the empirical performance of Merton (1974), Geske (1977), Longstaff and Schwartz (1995), Leland and Toft (1996) and Collin-Dufresne and Goldstein (2001) models using a sample of 182 bond prices during the period 1986-1997. For the Merton (1974) model the underestimation problem is confirmed but for other models, like Leland and Toft (1996), there is an overestimation of credit spread, which they report as due to the accuracy of the calibration process. The prediction power of these models seems to be related to leverage, size, asset volatility and some term structure control variables.

There are other empirical papers that calibrate some of the structural models of corporate bond pricing but that do not focus on the analysis of its performance. These include, among others, Anderson and Sundaresan (2000), Collin-Dufresne *et al* (2001), Huang and Huang (2002), Cooper and Davydenko (2003) and Vassalou and Xing (2004).

The reduced-form approach mentioned in the beginning of this section can also be seen as a way to overcome the problems found in structural models. By specifying the default process exogenously, it is possible to apply the reduce-form model to situations where the underlying asset value is not observable. In addition, since the default time is unpredictable, it is possible to capture the behaviour of credit spreads for short maturities more realistically.

A long list of papers has appeared recently in this field, but we highly Jarrow and Turnbull (1995) and Duffie and Singleton (1999). The first one presents a model where the bankruptcy process is compared to a spot exchange rate process and the default process is driven by a Poisson process with a constant intensity parameter and a given payoff at default. On the other hand, Duffie and Singleton (1999) demonstrate that valuation under the risk-neutral probability measure can be executed by discounting the non-defaultable payoff on the debt by a discount rate that is adjusted for the parameters of the default process.

Even though the reduced–form models reveal many attractive properties, they cannot establish the link between firm value and corporate default, as the structural models do.

2. Theoretical Models

In this section we describe the main theoretical assumptions of the Merton (1974), Leland (1994) and Fan and Sundaresan (2000) models. Moreover, there is a presentation of the formulas concerning the firm value, equity, debt and credit spread.

2.1 Merton (1974)

The Merton (1974) work, being the seminal paper of structural models, relies on a set of assumptions that constitute the basis for many other models. Most of them are embedded in the Black and Scholes option pricing theory. These assumptions can be summarized as follows:

Assumption 1: Markets are frictionless. There are no transaction costs, taxes, bankruptcy costs, agency costs or problems with indivisibility of assets.

Assumption 2: Every individual acts as if he can buy or sell as much of any security as he wishes without affecting the market price.

Assumption 3: There is a riskless asset, whose rate of return per unit of time is known and constant over time, implying a flat and constant term structure of risk free rates.

Assumption 4: Trading takes place continuously and individuals may take short positions in any security, including the riskless asset.

Assumption 5: The dynamics for the value of the assets, *Vt*, can be described by a diffusion-type process with stochastic differential equation

$$dV_t = (\mu - \delta)V_t \, dt + \sigma V_t \, dZ_t \tag{2.1}$$

where μ is the instantaneous expected rate of return on the assets, δ is the constant fraction of value paid to both equityholders and debtholders (payout ratio), σ the constant variance of the return on the underlying asset, and Z_t a standard wiener process. Even though the original version of Merton (1974) model assumes no payout ratio, we incorporate this parameter in our model, as most firms pay both interest to bondholders and dividends to equityholders.

Assumption 6: The asset value is financed both by equity, *E*, and one representative zero-coupon noncallable debt contract, *D*, with maturity *T* and face value *F*. Moreover, there are no issues of any type of security during the life of the debt contract.

Assumption 7: The absolute priority rule holds. At maturity, equityholders only obtain a positive payoff after debtholders being totally reimbursed.
Assumption 8: Even though the assets of the firm secure the debt, the debtholders cannot force the firm on bankruptcy until *T*.

From assumption 1 and 6 it follows that the value of the firm and the asset value are identical and do not depend on the capital structure itself. The asset value, V_0, is thus given by the sum of risky debt and equity.

$$V_0 = D + E \tag{2.2}$$

At maturity, in the case that the face value payment is not met, the bondholders receive the entire value of the firm (implying a recovery rate of 100%) and equityholders nothing. If the asset value is higher than the face value then, the equityholders, as residual claimers, receive the difference between these two values. This means that, at maturity, equity and debt are given, respectively, by:

$$E_T = \max\left[0, V_T - F\right] \tag{2.3}$$

$$D_T = \min\left[V_T, F\right] \tag{2.4}$$

With this framework, equity can be seen as a call option on the value of the firm with strike price *F*. On the other hand, debtholders have bought a risk free bond with face value *F* and given the equityholders the option to sell them the firm's assets for *F*. Equity value is therefore given by Black and Scholes (1973) formula

$$E_0(V_0, T, \sigma, r, F, \delta) = V_0 e^{-\delta T} N(d_1) - F e^{-rT} N(d_2) \tag{2.5}$$

with

$$d_1 = \frac{\ln\left(\frac{V_0}{F}\right) + \left(r - \delta + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

where $N(\circ)$ is the cumulative standard normal distribution.

As regards the debt value it is given by

$$\begin{aligned} D &= Fe^{-rT} - European\ Put \\ &= Fe^{-rT} - \left[Fe^{-rt}N(-d_2) - V_0 e^{-\delta T}N(-d_1)\right] \\ &= V_0 e^{-\delta T}N(-d_1) + Fe^{-rT}N(d_2) \end{aligned} \tag{2.6}$$

or alternatively:

$$D = V_0 - E \tag{2.7}$$

where d_1,d_2 and N(d_1) are defined above.

One of the most important variables that is analysed in this study is the credit spread, *CS*. It is defined as the difference between the yield to maturity, *ytm*, and the risk free rate *r*. The yield to maturity is the rate that makes equal the market value of debt to the present value of the face value of debt. It is computed as

$$ytm = -\frac{\ln\left(\frac{D}{F}\right)}{T} \tag{2.8}$$

Hence, the credit spread formula is

$$CS = ytm - r = -\frac{1}{T}\ln\left[N(d_2) + \frac{V_0}{Fe^{-rT}}N(-d_1)\right] \tag{2.9}$$

where it can be seen that the credit spread is a direct function of the quasi-debt ratio Fe^{-rT}/V_0, maturity and asset volatility. Intimately related to credit spread is the Risk Neutral Default Probability (RNDP), which is, in this case, represented by N(-d_2).

2.2 Leland (1994)

Leland (1994) introduces significant changes in relation to Merton's (1994) work. It assumes the possibility of early default and considers perpetual debt instead of a zero coupon debt. Moreover, Leland (1994) introduces taxes and bankruptcy costs into the debt pricing model, leading to the existence of an optimal capital structure.

Despite these new assumptions, the firm value follows (2.1) and the risk free rate is constant[2]. Leland (1994) models a tax environment in which continuous coupon payments, C, are tax deductible. Considering a constant corporate tax rate τ, the firm obtains tax shields from its debt at a rate $C\ \tau$ until default. Bankruptcy occurs when the firm value reaches a threshold V_b. In this case, the firm incurs costs αV_b, where α is defined as the bankruptcy cost parameter or one minus the recovery rate. Because of these new "real world features" the levered firm value, v, is no longer identical to the unlevered firm value V_u. Rather, the firm value increases in the amount of tax shield, TS, and decreases in the amount of bankruptcy cost, BC.

Under these new assumptions, the debt value is now

$$D = \frac{C}{r}\left(1 - P_b\right) + P_b\left(1-\alpha\right)V_b \tag{2.10}$$

where V_b is given by equation (2.11) and P_b is $\left(\frac{V_u}{V_b}\right)^{\lambda}$.

$$V_b = \frac{C\left(1-\tau\right)}{r}\frac{-\lambda}{1-\lambda} \tag{2.11}$$

The parameter λ in the bankruptcy trigger solution is

$$\frac{1}{2} - \frac{\left(r-\delta\right)}{\sigma^2} - \sqrt{\left[\frac{1}{2} - \frac{\left(r-\delta\right)}{\sigma^2}\right]^2 + \frac{2r}{\sigma^2}}$$

P_b can be interpreted as the risk neutral default probability in Leland's model and λ as the elasticity of the probability of default with respect to the value of the assets of the firm. As such, it is negative and increases with the volatility of the assets of the firm.

The bankruptcy costs are given by

$$BC = P_b \alpha V_b \tag{2.12}$$

[2] Once again we consider the version of Leland (1994) with payout ratio.

and the tax shield by

$$TS = \frac{\tau C}{r} - \frac{\tau C}{r}\left(\frac{V_u}{V_b}\right)^{\lambda} \quad (2.13)$$

The total firm value is defined as

$$v = E + D = V_u + TS - BC \quad (2.14)$$

leading to an equity value

$$E = v - D \quad (2.15)$$

Likewise, the credit spread is

$$CS = \frac{C}{D} - r \quad (2.16)$$

3.3 Fan and Sundaresan (2000)

Fan and Sundaresan (2000) debt-equity swap assumes that at an endogenously determined lower reorganization boundary debtholders are offered a proportion of the firm's equity to replace the original debt contract. This can be thought as a distress exchange. At a certain trigger point V_b the claimants negotiate not to operate the firm and sell their stake to outsiders who pay them the value of the assets of the firm. It resembles a swap because debtholders swap their debt for equity and then sell the equity to potential buyers. As the model assumes corporate taxes, there is the expectation that equity is priced properly to reflect the tax benefit of a future recapitalization. This tax advantage should offset the cost of a future renegotiation with outsiders.

Unlike Leland (1994), which does not include the possibility of debt renegotiation, Fan and Sundaresan (2000) assume a continuous bargaining power parameter η. When $\eta = 1$ equityholders have all the bargaining power and make take-it-or-leave-it offers to debtholders. On the other hand, when $\eta = 0$, we get Leland (1994) outcome where debtholders make take-it-or-leave-it offers to equityholders.

With this refinement in Leland’s (1994) model the valuation framework turns as follows. The debt value is now defined as

$$D = \frac{C}{r}\left(1 - P_b\right) + P_b\left(1 - \eta\alpha\right)V_b \tag{2.17}$$

where the new bankruptcy threshold is

$$V_b = \frac{C\left(1-\tau\right)}{r}\frac{-\lambda}{1-\lambda}\frac{1}{1-\eta\alpha} \tag{2.18}$$

and P_b and λ are defined as before.

Equity and firm value are given by equations (2.19) and (2.20), respectively.

$$E = V_u - \frac{C\left(1-\tau\right)}{r}\left(1 - P_b\right) + \eta\alpha V_b P_b - V_b P_b \tag{2.19}$$

$$v = E + D \tag{2.20}$$

As Fan and Sundaresan (2000) also assumes a continuous perpetual coupon, the credit spread is given by equation (2.16).

3. Conclusion

This paper surveys the theoretical and empirical literature on structural models of corporate debt pricing. During the last years we saw many theoretical developments in the field of credit risk research. Most of this research concentrated on the pricing of corporate and sovereign defaultable bonds as the basis of credit risk pricing. These studies can be divided in two main categories: structural models and reduced-form models.

Structural models have its origins in Merton (1974) framework, which has been the key foundation of corporate debt pricing. As noticed by Cossin and Pirrote (2001), “It is called the ‘structural approach’ because it relies entirely upon the sharing rule for the value of the assets of the firm between two main classes of claimholders, the shareholders and the bondholders, in other words, it depends on the actual capital structure of the firm”(p.15). In such a framework, the default process of a company is driven by the value of the company’s assets and the firm’s default risk is explicitly linked to the variability in the firm’s asset value. Under these structural models, all the relevant credit risk elements, including default, are a function of the structural characteristics of the firm: asset volatility (business risk) and leverage (financial risk).

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It provides an understanding of the importance of structural models in predicting credit spreads, and focuses on the role of rating, maturity, asset volatility and sector effects.

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